**Barron’s Let’s Review Regents – Algebra I**

# Chapter 2: Polynomic Arithmetic

## 2.1 Classifying Monomials, Binomials, and Trinomials

An expression like has two terms while has three terms. Expressions with only one term are called *monomials*. Expressions with two terms are called *binomials*. Expressions with three terms are called *trinomials*. They are all special types of *polynomials*.

### Determining the Type of a Polynomial by the number of terms

Example 1

6x - 2 has two terms and is a *binomial*.

Example 2

5x2 has no plus or minus signs, and is one term. It is a *monomial*.

### The Degree of a Polynomial

When the terms of a polynomial have exponents on the variables, the *degree* of a polynomial is the largest exponent that a variable is raised to. The binomial 2x5 + 3x is degree 5 since the largest exponent is 5. The trinomial 3x2 -7x + 3 is degree 2. A monomial like 7 is degree 0, since it is the equivalent of 7x0.

#### Math Facts

* A polynomial with a degree of 0 is called a *constant polynomial*.
* A polynomial with a degree of 1 is called a *linear polynomial*.
* A polynomial with a degree of 2 is called a *quadratic polynomial*.
* A polynomial with a degree of 3 is called a *cubic polynomial*.

### Check Your Understanding of Section 2.1

1. Multiple Choice
2. Classify the polynomial 5x2 + 3.  
   (2) Binomial
3. Classify the polynomial 7x2 -3 + 2.  
   (3) Trinomial
4. Classify the polynomial 3x3.  
   (1) Monomial
5. Classify the polynomial 3x2y + 5xy2.
6. Classify the polynomial 3x3 + 5x2 + 7x -3.  
   (4) None of the above.
7. Classify the polynomial 3x2 -7x + 5.  
   (3) Trinomial
8. Which of the following is a binomial?  
   (3) 5x2 + 2
9. Which of the following is a trinomial?  
   (3) 2x2 + 5x + 3
10. What is the degree of the polynomial 5x2 + 7x3 -2x?  
    (4) 3
11. Which of the following is a quadratic trinomial?  
    (3) 5x2 -3x + 8
12. Show how you arrived at your answers.
13. An expression for the height of a projectile is   
    -16t2 + 72t + 5. What kind of polynomial is this and what is its degree?  
      
    This has three terms and is a trinomial.  
    The highest exponent is 2. Therefore the degree is 2 and it is called a quadratic polynomial.
14. An equation for the amount of profit a company makes is 5q – 800. What kind of polynomial is that and what is its degree?  
      
    This is a binomial since it has two terms. The degree is one because its largest exponent is 1 (x1) it is called a linear polynomial.
15. Coss out one of the terms in this polynomial to make it into a third-degree trinomial:   
    5x3 + 2x2 -7x + 9.  
      
    5x3 + 2x2 -7x ~~+ 9~~
16. Create a fourth-degree monomial.  
      
    5x4
17. Mark says that the expression 2 + 3 is a binomial since it has two terms. Layla says that it is a monomial since 2 + 3 = 5, and 5 is just one term. Who is correct?  
      
    Technically 2 + 3 is a binomial since it has two terms. 5 is a monomial since it has only one term. Layla is slightly incorrect when disputing it being a binomial.

## 2.2 Multiplying and Dividing Monomials

A monomial, like 5x3, has a coefficient of 5 and a variable part of x3. Just as numbers can be multiplied together, monomials can be multiplied. When multiplying monomials, you have to use the rules for exponents.

What is a coefficient? A coefficient is a number that is multiplied by a variable.

### Multiplying Expressions Involving Exponents

To multiply two monomials, multiply the two coefficients, and multiply the variable parts using the exponent multiplication shortcut of adding the exponents.

**Example 1**

Simplify x2 \* x3 : x5

### Multiplying Monomials That Have Coefficients and Variable Parts

**Example 2**

Simply 3x2 \* 5x3   
(3\*5) \* x2 \* x3 = 15x5

**Example 3**

Simplify 3x4 \* 4x = (3 \* 4) \* x4 \*x = 12x5

**Example 4**

Simplify 2x5 \* 3x4 \* 5x3 = (2\*3\*5) \* x5+4+3 = 30x12

**Example 5**

Simplify 4x2y \* 7xy3 = (4\*7) \* (x2 \* x) \* (y \* y3) =  
28x3y4

**Math Facts**

### Dividing Monomials

To divide two monomials, divide the two coefficients and subtract the exponent of the divisor from the exponent of the dividend.

Example 6

Simplify

### Check Your Understanding of Section 2.2

1. Multiple Choice
2. Multiply 3x3 \* 4x5(3) 12x8
3. Multiply 2x5 \* 5x  
   (1) 10x6
4. Which of the following would not simplify to 12x6?  
   (4) 4x3 \* 3x2
5. Which of the following would not simplify to 24x5?  
   (4) 6x5 \* 4x
6. Multiply x6 \* x.  
   (3) x7
7. Multiply 5 \* 3x5.  
   (2) 15x5
8. If 2x3y = 10x6, which expression is equivalent to y?  
   (2) 5x3
9. Multiply 3x5 \* 2x-2.  
   (3) 6x3
10. If 5x7z = 20x4, which expression is equivalent to z?  
    (4) 4x-3
11. Simplify (4x3)2.  
    (1) 16x6
12. Show how you arrived at your answers.
13. Alexa multiplies 4x2 \* 5x3 and saysthe answer is 20x6 since 4 \* 5 is 20 and 2 \* 3 is 6. Is she correct?  
      
    Coefficients are multiplied and exponents are added for the variable x. The correct answer is **20x5.**
14. What does 12x6 3x2 simplify to?  
    12x6 3x2 = (12 / 3) \* x6-2 = **4x4**
15. Joseph says that x0 = 0 since there are no x’s multiplied together. Sawyer says this is not correct. Who is right?  
      
    Sawyer is correct, because according to the “zero exponent rule”, any variable raised to the zeroth power is equal to 1 (one).
16. What is 5x2  2x-2? Explain your reasoning.  
      
    Coefficients are multiplied and exponents are added when simplifying monomials using the same variable.
17. When multiplying a quadratic monomial by a cubic monomial, what type of polynomial will result?  
      
    A quadratic monomial has degree 2 and a cubic monomial has degree 3. Therefore the multiplication will produce a **polynomial of degree 5**.

## 2.3 Combining Like Terms

Two terms are called *like terms* if they have the same variable part. For example 3x2 and 5x2 are like terms since they both have an x2 as the variable part. 3x2 and 5x3 are not like terms because the x2 is different from the x3. Like terms can be combined with addition or subtraction.

### Combining Two Like Terms

When you add two dogs to three dogs, you get five dogs. Likewise, 2x + 3x = 5x, 2x2 + 3x2 = 5x2, and 2x10 + 3x10 = 5x10. For example 2x + 3y cannot be simplified any further. Event though 2x + 3x2 each have a variable part containing x, they are not like terms since the variable part must be identical.

### If There Is No Coefficient, It Is Really a Coefficient of 1

5x2 + x2 is the same as 5x2 + 1x2.

**Example 1**

4x3 + 6x3 = 10x3

**Example 2**

6x3 – 2x2 = 6x3 – 2x2

**Example 3**

5x2y3 + 4x2y3 = 9x2y3

### Check Your Understanding of Section 2.3

1. *Multiple Choice*
2. Which expression is equivalent to 2x2 + 5x2?  
   (2) 7x2
3. Which expression is equivalent to 3x4 + 5x?  
   (4) The expression cannot be simplified any further.
4. Which expression is equivalent to -5x3 + 2x3?  
   (3) -3x3
5. Which expression is equivalent to   
   8y2 -10y2 + 5y2?  
   (3) 3y2
6. Which expression is equivalent to 5xy2 + 3xy2 – 4x2y?  
   (4) 8xy2 – 4x2y
7. Which expression is *not* equivalent to 7x5?  
   (1) 5x2 + 2x3
8. Which expression is equivalent to 3x2?  
   (2) x2 + 2x2
9. Which expression is *not* equivalent to 7x2y?  
   (4) 2 + 5x2y
10. Which expression cannot be simplified any further?  
    (3) 5x + 3x2
11. Which expression is equivalent to   
    x2 + x2 + x2 + x2?  
    (4) 4x2
12. *Show how you arrived at your answers*.
13. Katherine simplifies the expression 3x2 + 5x2 by noticing that x2 (3 + 5) would become 3x2 + 5x2 if she used the distributive property of multiplication over addition but would become x2(8) if she simplified the parentheses first. Was this a valid way to arrive at the correct answer 8x2? Explain your reasons. Refactoring using the distributive property of multiplication over addition, and then simplifying is a correct way to simplify. Combining Like Terms using addition of coefficients is essentially the same process.
14. Chelsea says the express 3xy2 + 5y2x cannot be simplified any further since the terms are not like terms. Her friend says that it’s possible. Who’s right? Explain your reasoning.  
      
    Her friend is correct. Using the commutative property of multiplication, and using like terms:  
    5y2x = 5xy2  
    3xy2 + 5y2x = 3xy2 + 5xy2 = 8xy2
15. If 5xy2z + a = 7xy2z, what expression in terms of x, y and z must the variable a be equivalent to?  
      
    Using the addition property of equality:  
    5xy2z + a = 7xy2z  
    -5xy2z = -5xy2z  
    a = 2xy2z
16. Combine all like terms in the expression below:  
    x3 + x2y + xy2 + y3 + x2y + xy2 + xy2 + x2y

x3 + y3 + x2y + x2y + x2y + xy2 + xy2 + xy2

x3 + y3 + 3x2y + 3xy2

1. Combine all like terms in the expression:  
   6 – 3i + 2i – i2.  
     
   6 – 3i + 2i – i26 – i – i2

## 2.4 Multiplying Monomials and Polynomials

### Multiplying Monomials by Binomials

To multiply the monomial 2x2 by the binomial 3x + 5, put the 3x + 5 in parentheses and write the problem as 2x2 (3x + 5). Now distribute the 2x2 to each of the terms in the parentheses to get 2x2 3x + 2x2 5. This can be simplified to 6x3 + 10x2.

### Multiplying Monomials by Polynomials with More than Two Terms

The distributive property applies even when there are more than two terms in the parenthesis. 5(1 + 2 + 3) = 5\*1 + 5\*2 + 5\*3 = 5 + 10 + 15 = 30.

To multiply 5x by 2x2 + 7x – 3, multiply the 5x by each of the three terms in the second expression and then simplify the terms if possible.

5x (2x2 + 7x – 3) = 10x3 + 35x2 -15x

### Check Your Understanding of Section 2.4

1. Multiple Choice
2. Multiply 5 by 2x + 3.  
   (2) 10x + 15
3. Multiply -2 by 5x2 -2x + 6.  
   (4) -10x2 + 4x -12
4. Multiply 3x2 + 7x - 5 by 4x2.  
   (3) 12x4 + 28x3 -20x2
5. Simplify 6x (2x + 3).  
   (2) 12x2 + 18x
6. Simplify -2x (3x -5).  
   (4) -6x2 + 10x
7. Simplify -4x (-2x – 7).  
   (4) 8x2 + 28x
8. Simplify 3x (5x2 - 2x + 3)  
   (2) 15x3 – 6x2 + 9x
9. Simplify 4x2y (3xy – 2xy2)  
   (3) 12x3y2 – 8x3y3
10. If a (2x + 5) = 8x2 + 20x, which expression is a equivalent to?  
    (1) 4x
11. Which expression is *not* equivalent to   
    2x3 + 10x2 – 6x?  
    (2) 2x2 (x2 + 5x -6)
12. Show how you arrived at your answers.
13. Tucker does the question 5 (2x + 3x) by first combining the like terms and gets 5(5x) = 25x. Alexander does the same question by applying the distributive property. Show how Tucker completes the question, and determine if Alexander gets the same answer as Tucker.  
      
    5 (2x + 3x) = 10x + 15x = 25x  
    Yes, Alexander gets the same answer as Tucker.
14. Jack says that:  
    (3x2 + 2x + 1) 5x = 15x3 + 10x2 + 5x.   
    His friend Izabella says that the monomial has to be on the left of the parentheses to do it this way. Who is correct and why?  
      
    Jack is correct. The commutative property of multiplication says that switching the order of the multiplicand and multiplier does not change the value of the expression.
15. The polynomial 10x3 – 6x2 + 14x can be written as a(5x2 – 3x + 7). What expression must the variable *a* represent? Explain your reasoning.  
      
    a = 2x  
    2x(5x2 – 3x + 7) = 10x3 - 6x2 + 14x
16. Simplify and write as a trinomial   
    4x (2x + 3) – 2(2x + 3) = 8x2 + 12x – 4x – 12 =  
    8x2 + 8x – 12
17. Simplify and write as a trinomial   
    2x(3x – 5) – 6(3x – 5) = 6x2 – 10x - 18x + 30 =  
    6x2 – 28x + 30

## 2.5 Adding and Subtracting Polynomials

### Adding Polynomials

**Example 1**

Simply the expression (2x + 3) + (3x + 5)  
**5x + 8**

**Example 2**

Simplify the expression   
(x2 + 3x + 2 + (x2 – 5x + 7).  
**2x2 - 2x + 5**

### Subtracting Polynomials

Most common errors: not distributing the negative sign through the parentheses.

**Example 3**

Simplify the expression (8x + 5) - 3(3x + 2)  
8x + 5 – 9x -6 = **-x – 1**

**Example 4**

Simplify the expression: (5x – 2) – (2x + 7)  
5x – 2 -2x – 7 = **3x – 9**

**Example 5**

Simplify the expression (7x + 3) – (4x – 5)

7x + 3 – 4x + 5 = **3x + 8**

### Check Your Understanding of Section 2.5

1. Multiple Choice
2. Simplify (2x + 3) + (4x + 5)  
   (1) 6x + 8
3. Simplify (3x – 2) + (6x + 5)  
   (1) 9x + 3
4. Simplify (x2 + 3x + 5) + (x2 + 4x - d2).  
   (2) 2x2 + 7x + 3
5. Simplify (x2 – 6x + 4) + 3(x2 + 2x – 5)  
   (x2 – 6x + 4) + 3x2 + 6x -15  
   (3) 4x2 – 11
6. Simplify (5x + 3) – (3x + 2)  
   5x + 3 -3x – 2 =   
   (4) 2x + 1
7. Simplify (3x – 4) – (5x – 3)  
   3x – 4 – 5x + 3 =  
   (2) -2x – 1
8. Simplify 2x(3x + 4) – 4x(2x – 5)   
   6x2 + 8x - 8x2 + 20x  
   (1) -2x2 + 28x
9. If C = 2x2 + 3x – 2 and D = -4x2 – 3x + 5, then C – D =  
   (2x2 + 3x – 2) – (-4x2 – 3x + 5) =  
   (2x2 + 3x – 2) + (4x2 + 3x - 5) =  
   (3) 6x2 + 6x – 7
10. Simplify (3x2 – 4x + 2) - 2(x2 – 2x + 1)  
    (3x2 – 4x + 2) + (-2x2 + 4x – 2)  
    (1) x2
11. Simplify (x2 – 5x + 6) – (x2 – 2x + 1).  
    (x2 – 5x + 6) + (-x2 + 2x - 1) =  
    (3) -3x + 5
12. Show how you arrived at your answers.
13. Nathan tried to simplify the expression (2x + 3) – (4x + 2) by first writing   
    2x + 3 - 4x + 2 and then simplifying to   
    -2x + 5. Did he do this correctly? Explain your reasoning.  
      
    No. One of the most common errors is to not distribute the minus sign through all the terms in the expression,  
    Correction: (2x + 3) + (-4x – 2) = -2x + 1
14. If C = 3x + 5 and D = 2x - 5, determine   
    C - 3D and simplify as much as possible.  
      
    C – 3D = (3x + 5) - 3(2x – 5) =  
    (3x + 5) – 6x + 15 =   
    **-3x + 20**
15. To solve 49 – 2(13), Kaleigh changed it into (40+9) – 2(10+3) and then simplified to 40 + 9 – 20 – 6 = 20 + 3 = 23. Explain why this produces the right answer.  
      
    49 = 40 + 9  
    13 = 10 + 3  
    The minus sign is distributed correctly over the terms in the expression:   
    -2(10 + 3) = -20 – 6  
    The math is then done correctly.
16. If F = -2x + 5 and G = 3x – 7, express   
    2F – 5G in terms of x and y simplified as much as possible.  
    2F – 5G = 2(-2x + 5) – 5(3x – 7) =  
    -4x + 10 -15x + 35 =  
    **-19x + 45**
17. If P = x2 + 5x + 2, Q = x2 – 3x + 4, and   
    R = x2 - 2x + 5, what is P + Q – R in terms of x simplified as much as possible?  
      
    P + Q – R =   
    (x2 + 5x + 2) + (x2 – 3x + 4) - (x2 - 2x + 5) =   
    2x2 + 2x + 6 + (-x2 + 2x – 5) =  
    **x2 + 4x + 1**

## 2.6 Multistep Algebra Equations Involving Polynomial Arithmetic

### Simplifying One Side of the Equation Before Solving

**Example 1**

Solve for x in the equation 5(x+3) – 2x = 7.  
5(x+3) – 2x = 7  
5x + 15 - 2x = 7  
3x = 7 – 15 = -8

### Solving an Equation with Variable on Both Sides of the Equals Sign

**Example 2**

5x + 3 = 3x + 15  
5x – 3x = 15 – 3  
2x = 12  
x = 6

### Word Problems Involving Combining Like Terms

**Example 3**

Angelina has x dollars. Alexander has 5 more than 3 times the amount of money that Angelina has. Together they have 41 dollars. Hom much does each have?

x + (3x + 5) = 41  
4x = 41 – 5 = 36   
**x = 9 (Angelina)  
Alexander has (3\*9) + 5 = 32 dollars**  
9 + 32 = 41

**Example 4**

In five years Gabriel will be three years less than twice his current age.

x = Gabriel’s current age

(x + 5) = 2x – 3  
5 + 3 = 2x – x  
**x = 8**

### Check Your Understanding of Section 2.6

1. Multiple Choice
2. Solve for x:   
   (2x + 3) + (4x + 5) = 32  
   2x + 4x + 3 + 5 = 32  
   6x + 8 = 32  
   6x = 32 – 8 = 24  
   **(1) x = 4**
3. Solve for x:   
   (5x + 7) – (2x + 3) = 25  
   5x + 7 – 2x – 3 = 25  
   3x + 4 = 25  
   3x = 21  
   (3) x = 7
4. Solve for x:  
   x + 3x – 5 = 195  
   4x = 195 + 5 = 200  
   **(3) x = 50**
5. Solve for y: y + (y + 1) + (y + 2) = 45  
   3y + 3 = 45  
   3y = 42  
   **(1) y = 14**
6. Solve for x:   
   5x + 50 = 7x  
   50 = 2x  
   **(4) x = 25**
7. Solve for z:   
   30z = 300 – 20z  
   50z = 300  
   **(4) z = 6**
8. Solve for a:  
   2a + 5 = 4a – 1  
   6 = 2a  
   (2) a = 3
9. Solve for x:   
   x2 + 4x + 6 = x2 – 2x + 18  
   -x2 + 2x – 6 = -x2 + 2x - 6  
   6x = 12  
   **(3) x = 2**
10. The width of a rectangle is 5 inches more than its length. The perimeter of the rectangle is 58 inches. Which equation could be used to find the length of the rectangle?  
    x = width, y = length  
    x + 5 = y  
    2x + 2y = 58  
    **(1) 2x + 2(x + 5) = 58**
11. Jonathan weighs thirty pounds more than three times his son Aidan’s weight. If their combined weight is 210 pounds, which equation could be used to find the weight of Aidan?  
    x = Aidan’s weight  
    **(1) x + 3x + 30 = 210**
12. Show how you arrived at your answers.
13. In 20 years, Lillian will be three years older than twice her current age. Set up an equation that can be used to solve for x, Lillian’s current age.  
    2x + 3 = x + 20  
    x = 17
14. The equation 2x + 5 = 6x – 7 can be solved several ways. Makayla solves it first by subtracting 2x from both sides of the equation. David solves it by first subtracting 6x from both sides. Which method would you choose and why?  
    2x + 5 = 6x – 7  
    -2x = -2x  
    5 = 4x – 7  
    4x = 12  
    x = 3  
    I prefer dealing with positive x coefficients rather than negative x coefficients.
15. Solve for x in 2(x + 3) – 5(x – 2) = 4.  
    2x + 6 – 5x + 10 = 4  
    -3x + 16 = 4  
    -16 = -16  
    -3x = -12  
    x = 4
16. Two trains are 300 mile away from each other on the same set of tracks. The train goes east at a speed of 30 miles per hour. The second train goes west at 20 miles per hour. An equation that can be used to determine how long for the trains to pass is 30x = 300 – 20x. Explain how this equation was formed, and solve for x to determine when the trains will pass.  
    x = number of hours when the trains will pass  
    Train traveling east will go 30x in x hours.  
    Train traveling west will go 300 – 20x in x hours.  
    When they meet, the distance will be the same.  
    30x = 300 – 20x  
    50x = 300  
    x = 6 hours  
    Train traveling east will travel 180 miles.  
    Train traveling west will travel 120 miles and be at mile marker 300 – 120 = 180 miles when the trains meet.
17. On Tuesday, an electronics store sells five items less than double the amount it sold on Monday. On Wednesday, the store sells ten items more than the amount it sold on Monday. If it sold 125 items together on the three days, create an equation and use it to determine the number of items sold each day.  
    x = # of items sold on Monday  
    y = # of items sold on Tuesday  
    z = # of items sold on Wednesday  
      
    y = 2x – 5  
    z = x + 10  
    x + y + z = 125  
    x + (2x – 5) + (x + 10) = 125  
    x + 2x + x – 5 + 10 = 125  
    4x + 5 = 125  
    4x = 120  
    **x = 30  
    y = (2 \* 30) – 5 = 60 – 5 = 55  
    z = 30 + 10 = 40**  
    30 + 55 + 40 = 125

## 2.7 Multiplying Polynomials by Polynomials

Multiplying polynomials can be done by applying the distributive property several times and then combining like terms.

**Multiplying Binomials with the Distributive Property**

(x + 2)x + (x + 2)5 = x2 + 2x + 5x + 10 = x2 + 7x + 10

**Example 1**

Multiply (x +4)(x – 3)   
(x +4)(x – 3) = xx – 3x + 4x -12 = **x2 + x – 12**

**Multiplying Binomials with the FOIL Shortcut**

1. **F – Firsts**
2. **O – Outers**
3. **I – Inners**
4. **L - Lasts**

(x - 6)(x + 2) = xx + 2x + (-6)(x) + (-6)(2) =  
x2 + 2x – 6x – 12 = **x2 - 4x – 12**

**Example 2**

Multiply (x + 2)(x + 5)  
(x + 2)(x + 5) = xx + 5x + 2x + 10 =   
**x2 + 7x + 10**

**Example 3**

Multiply (x + 4) (x – 2)  
xx + (-2)(x) + (4)(x) + (4)(-2) = x2 - 2x + 4x – 8 =  
**x2 + 2x – 8**

**Example 4**

Multiply (2x – 3)(5x + 2).  
(2x)(5x) + (2x)(2) + (-3)(5x) + (-3)(2)  
10x2 + 4x – 15x – 6 = **10x2 – 11x – 6**

**Example 5**

Multiply (x + 5)(x + 5)  
xx + (x)(5) + (5)(x) + (5)(5) = x2 + 5x + 5x + 25 =  
**x2 + 10x + 25**

### Squaring Binomials

Example 6

Simplify (x + 6)2

(x + 6)(x + 6) = xx + (x)(6) + (6)(x) + (6)(6) =  
x2 + 6x + 6x + 36 =   
**x2 + 12x + 36**

### Multiplying the (a – b)(a + b) Pattern

The answer will be a2 – b2, because the middle terms will cancel out.

**Example 7**

Multiply (x + 5)(x – 5)  
xx + (x)(-5) + (5)(x) + (5)(-5) = x2 – 5x + 5x – 25 =   
**x2 – 25**

### Multiplying Polynomials by Polynomials

**Example 8**

Multiply (x + 3)(x2 + 5x + 6)

xx2 + (x)(5x) + (x)(6) + (3)(x2) + (3)(5x) + (3)(6) =  
x3 + 5x2 + 6x + 3x2 + 15x + 18 =   
**x3 + 8x2 + 21x + 18**

### Check Your Understanding of Section 2.7

1. Multiple Choice
2. (x + 2)(x + 7) = (x)(x) + (x)(7) + (2)(x) + (2)(7) =  
   x2 + 7x + 2x + 14 =   
   **(3) x2 + 9x + 14**
3. (x + 2)(x – 7) = (x)(x) + (x)(-7) + (2)(x) + (2)(-7) =  
   x2 – 7x + 2x – 14 =   
   **(1) x2 - 5x – 14**
4. (2x + 3)(3x – 1) =   
   (2x)(3x) + (2x)(-1) + (3)(3x) + (3)(-1) =  
   6x2 – 2x + 9x – 3 =   
   **(3) 6x2 – 7x – 3**
5. (2a + 5b)(4a + 3b) =   
   (2a)(4a) + (2a)(3b) + (5b)(4a) + (5b)(3b) =  
   8a2 + 6ab + 20ab + 15b2 =   
   **(4) 8a2 + 26ab + 15b2**
6. (x + 5)2 = (x + 5)(x + 5) = x2 + 5x + 5x + 25 =   
   **(4) x2 + 10x + 25**
7. (x – 7)(x + 7) = (x)(x) + (x)(7) + (-7)(x) + (-7)(7) =  
   x2 + 7x – 7x -49 =   
   (2) x2 – 49
8. (x2 – 2)(x2 + 6) =  
   (x2) (x2) + (x2)(6) + (-2) (x2) + (-2)(6) =  
   x4 + 6x2 - 2x2 – 12 =  
   **(1) x4 + 4x2 – 12**
9. (x2 – 4)(x2 – 9) =   
   (x2) (x2) + (x2) (-9) + (-4) (x2) + (-4) (-9) =  
   x4 - 9x2 - 4x2 + 36 =   
   **(2) x4 - 13x2 + 36**
10. If (x + 4)(x + a) = x2 + 7x + 12, what is the value of a?  
    (x)(x) + (x)(a) + (4)(x) + 4a =   
    x2 + ax + 4x + 4a = x2 + 7x + 12  
    **(3) a = 3**
11. A 4-foot by 6-foot patio has a 2-foot border around it. The area of the combined patio and border can be expressed as (2x + 4)(2x + 6). Which expression is equivalent to this?  
    (2x + 4)(2x + 6) =   
    (2x)(2x) + (2x)(6) + (4)(2x) + (4)(6) =   
    4x2 + 12x + 8x + 24 =   
    **(2) 4x2 + 20x + 24**
12. Show how you arrived at your answers.
13. If C = x + 4 and D = x – 1, express CD + 2C as trinomial.  
    CD + 2C = (x + 4)(x – 1) + 2(x + 4) =  
    (x)(x) + (x)(-1) + (4)(x) + (4)(-1) + 2x + 8 =   
    x2 – x + 4x – 4 + 2x + 8 =   
    **x2 + 5x + 8**
14. Express (2 + 3i)2 + (1 + 2i) as a trinomial.  
    (2 + 3i)(2 + 3i) + (2i + 1) =  
    (2)(2) + (2)(3i) + (3i)(2) + (3i)(3i) + (2i + 1) =  
    4 + 6i + 6i +9i2 + 2i + 1 =   
    **9i2 + 14i + 5**
15. Maria says that (x + 4)2 = x2 + 16 because of the distributive property. Is she correct? Explain the reasoning.  
    **There is a distributive property of multiplication, but no distributive property for raising to a power.**(x + 4)(x + 4) = (x)(x) + (x)(4) + (4)(x) + (4)(4) =  
    x2 + 4x + 4x + 16 =   
    **x2 + 8x + 16**
16. What is (x + 3)(x2 + 7x + 10) expressed as a four-term polynomial?  
    (x)(x2)+(x)(7x)+(x)(10)+(3)(x2)+(3)(7x)+(3)(10) =  
    x3 + 7x2 + 10x + 3x2 + 21x + 30 =  
    **x3 + 10x2 + 31x + 30**
17. Noah says that he has a shortcut for multiplying 4852 in his head. He starts by writing 48 as 50 – 2 and 52 as 50 + 2. Explain how he can complete this question.  
    He can use FOIL to multiply all the terms.  
      
    4852 = (50 – 2)( 50 + 2) =  
    (50)(50) + (50)(2) + (-2)(50) + (-2)(2) =  
    2500 + 100 – 100 – 4 = 2496

## 2.8 Factoring Polynomials

Factoring a number is when you find two numbers whose product is that number. For example, the number 15 can be factored into 35. Some polynomials can be factored by reversing the multiplication processes from the previous sections.

### Greatest Common Factor Factoring

When you use the distributive property to multiply an expression like 4(3x + 5), you get 12x + 20. Factoring 12x + 20 requires undoing the distributive property. Examine the terms 12x and 20 and determine if they have a common factor.

4 ( ) = 12x + 20

**Example 1**

Factor the trinomial 2x2 – 8x + 10.

2 ( ) = 2x2 – 8x + 10  
2 (x2 – 4x + 5) = 2x2 – 8x + 10

**Example 2**

Factor the trinomial 12x3 + 18x2 - 24x

6x ( ) = 12x3 + 18x2 - 24x  
6x ( 2x2 + 3x – 4) = 12x3 + 18x2 - 24x

### Factoring a Trinomial into the Product of Two Binomials

When you multiply (x + 2) and (x + 5), you get   
x2 + 7x + 10.The coefficient of the x term is 7 and the constant is 10. Notice that 5 and 2 are the two constants of the binomial and that 7 = 5 + 2 and   
10 = 5.

**Example 3**

Factor x2 + 8x + 15 into the product of two binomials.  
35 = 15, 3 + 5 = 8.  
**(x + 3)(x + 5)** = (x)(x) + (x)(5) + (3)(x) + (3)(5) =  
x2 + 5x + 3x + 15 = x2 + 8x + 15

**Example 4**

Factor x2 - 3x -18 into the product of two binomials.  
The six factor pairs of -18 are (-1)(18), (1)(-18), (-2)(9), (2)(-9), (-3)(6), and (3)(-6).  
(-6)(3) = -18, -6 + 3 = -3.  
**(x – 6)(x + 3)** = (x)(x) + (x)(3) + (-6)(x) + (-6)(3) =  
x2 + 3x – 6x – 18 = x2 – 3x -18.

**Example 5**

Factor 3x2 + 30x + 63  
3x2 + 30x + 63 = (3)(x2 + 10x + 21)  
  
Factor Pairs: (1)(21), (3)(7). Both must be positive or both must be negative.  
(3)(7) = 21, 3 + 7 = 10  
W  
(3)(x + 3)(x+7) = (3) ( (x)(x) + (x)(7) + (3)(x) + (3)(7) ) =  
(3) ( x2 + 7x + 3x + 21) = (3) (x2 + 10x + 10)

### Factoring a Perfect Square Trinomial into the Square of a Binomial

When you multiply a number by itself, like 3 times 3, the result is 9, which is known as a square number because it can be factored into 32.

When you use FOIL to square a binomial, like (x +5)2, you get x2 + 10x + 25. This is known as a perfect s square trinomial, because it can be factored back into (x +5)2.  
  
All perfect square trinomials x2 + bx + c have property that and factor into .

**Example 6**

Is x2 + 12x + 36 a perfect square trinomial?

Yes. If you calculate 2 you get 36, which is equal to the constant term.

**Example 7**

Is x2 – 6x + 8 a perfect square trinomial?

No. 2 = 9, not 8.

**Example 8**

If x2 - 14x + 49 a perfect square trinomial, factor it.  
  
2 = 2 = (-7)2 = 49. Yes. This is a perfect square trinomial.

Factors: (7)(7) = 49  
**(x – 7)2 = (x – 7)(x – 7)** = (x)(x) + (x)-7) + (-7)(x) + (-7)(-7) =  
x2 -7x -7x + 49 = x2 -14x + 49

### Difference of Perfect Squares Factoring

When you multiply the binomials (x – 5)(x + 5) with FOIL, you get x2 + 5x – 5x – 25 = x2 – 25. Notice that x2 and 25 are both perfect squares.

The rule is that x2 – b2 = (x – b)(x + b).

**Example 9**

If possible, factor x2 – 16.  
Both x2 and 16 are perfect squares.  
  
x2 – b2 = (x – b)(x + b).  
x2 – 16 = x2 – 42 = (x – 4)(x + 4)

**Example 10**

If possible, factor x2 + 49.  
This is not a perfect square because it is not of the form: x2 – b2.

**Example 11**

If possible, factor 3x2 – 48.  
  
3x2 – 48 = (3)( x2 – 16) = (3)(x2 – 42) = (3)(x – 4) (x + 4)

### Test-Taking Strategy for Multiple-Choice Factoring Questions

**Example 12**

Which choice is a factorization of x2 – 3x – 180?

(x + 12)(x – 15) = (x)(x) + (x)(-15) + (12)(x) + (12)(-15) =  
x2 - 15x + 12x – 180 = x2 -3x – 180.  
  
(2) (x + 12)(x – 15)

### Check your understanding of Section 2.8

1. Multiple Choice
2. Factor 3x2 + 6x + 15.  
   **(4) 3(x2 + 2x + 5)**
3. Factor completely 2x3 – 8x2 + 14x.  
   **(3) (2x)(x2 – 4x + 7)**
4. Factor x2 + 10x + 16.  
   Factors: (2)(8), (4)(4)  
   **(1) (x + 2)(x + 8)**
5. Factor x2 – 2x – 15  
   (Factors: (5)(-3), (3)(-5)  
   **(2) (x + 3)(x – 5)**
6. Factor x2 + x – 42.  
   Factors: (21)(-2), (-21,2), (7,-6), (-7, 6)  
   **(2) (x – 6) (x + 7)**
7. Factor x2 + 8x + 16.  
   Factors (2,8), (4,4)  
   **(1) (x + 4)2**
8. Factor x2 – 12x + 36.  
   Factors: (-6)(-6), (-9, -4), (-12, -3)  
   **(1) (x – 6)2**
9. Factor x2 – 100.  
   Perfect Square: x2 - 102  
   **(1) (x – 10)(x + 10)**
10. Factor completely 5x2 – 20.  
    (5)(x2 – 4) =   
    **(4) (5)(x – 2)(x + 2)**
11. Factor completely 6x2 + 12x – 144  
    (6) (x2 + 2x – 24) =   
    **(2) (6)(x – 4)(x + 6)**
12. Show how you arrived at your answers.
13. Kenzie says that x2 + 25 can be factored into (x + 5)2. Ariana says that it cannot. Who is correct. Explain your answer.  
      
    **Ariana is correct. Kenzie is incorrect.**(x + 5)2= (x + 5)(x + 5) =   
    (x)(x) + (x)(5) + (5)(x) + (5)(5) =  
    x2 + 5x + 5x + 25 = x2 + 10x + 25.
14. Factor completely x3 + 4x2 – 21x.  
    x3 + 4x2 – 21x = (x)(x2 + 4x – 21) =  
    **(x)(x + 7)(x – 3)**
15. Factor completely 5x2 + 20x + 20.  
      
    5x2 + 20x + 20 = (5)(x2 + 4x + 4) =  
    **(5)(x + 2)2**
16. Korbin says that there are two different ways to factor x2 – 5x + 6. One is (x – 2)(x -3) and the other is (x – 6)(x – 1). Is he correct? Explain your reasoning.  
      
    No. Korbin is not correct. Only the first answer is correct.  
      
    (x – 2)(x – 3) = (x)(x) + (x)(-3) + (-2)(x) +(-2)(-3) =  
    x2 – 3x – 2x + 6 = x2 – 5x + 6 (correct)  
      
    (x – 6)(x – 1) = (x)(x) + (x)(-1) + (-6)(x) + (-6)(-1) =  
    x2 – x – 6x + 6 = x2 – 7x + 6 (incorrect).
17. The polynomial 2x2 + 13x + 15 can be factored into (2x + 3) multiplying by another factor. What is the other factor? Explain your reasoning?  
      
    35 = 15  
    25 + 3 = 13  
      
    (2x + 3)(x + 5) = (2x)(x) + (2x)(5) + (3)(x) + (3)(5)  
    2x2 + 10x + 3x + 15 = 2x2 + 13x + 15

### 2.9 More Complicated Factoring

Some polynomials don’t, at first, appear to match any of the factoring patterns.

The three most frequent patterns are factoring of a trinomial into two binomials, the difference of perfect squares, and the perfect square trinomial pattern.

**Recognizing the Difference of Perfect Squares Pattern**

If there are only two terms and they are separated by a minus sign, there is a chance it will be possible to eventually use the difference of perfect squares pattern.

**Example 1**

Factor 9x2 – 4   
9x2 = (3x)24 = 22**9x2 – 4 = (3x + 2)(3x – 2)**

**Example 2**

Factor x4 – 9

x4 = (x2)29 = 32

x4 – 9 = **(x2 + 3)(x2  - 3)  
  
Note: (x2  - 3)**x2 = (x)2  
3 =   
(x2  - 3) = **(x + )(x - )**

**Example 3**

Completely factor x4 – 81

x4 = (x2)2  
81 = 92

x4 – 81 = (x2 + 9)( (x2 – 9)  
x2 = (x)2  
9 = 32  
x4 – 81 = (x2 + 9)( (x2 – 9) =  
**(x2 + 9)( (x+ 3)(x – 3)**

**Recognizing the Factoring of a Trinomial into Two Binomials Pattern**

If a polynomial has three terms where the exponent of one of the terms is half the largest exponent and one of the terms is a constant, it can sometimes be factored.

**Example 4**

Factor x4 + 4x2 – 12  
Factors (-2,6), (2,6), (-3,4), (-4,3)  
6 – 2 = 4  
x4 + 4x2 – 12 = **(x2 – 2)(x2 + 6)**

**Note: x2 – 2 could be further factored as the difference of perfect squares pattern.**

**Example 5**

Factor the expression of x4 + 8x2 - 9 completely.  
Factors: (-1, 9), (1, -9), (-3, 3)

x4 + 8x2 - 9 = (x2 – 1) (x2 + 9) **= (x + 1)(x -1) (x2 + 9)**

Recognizing the Perfect Square Trinomial Pattern

If there is a trinomial where one of the exponents is half of another exponent and one term is a constant, it will be a perfect square trinomial when the square of the coefficient of the term with the smaller exponent is equal to the constant).

Example 6

Factor x6 + 8x3 + 16  
16 = 42

= 42 = 16

x6 + 8x3 + 16 = (x3 + 4)2

### Check Your Understanding of Section 2.9

1. Multiple Choice
2. Factor (2x)2 – 25  
   (2x)2 – 25 = 4x2 – 25 =   
   **(3) (2x – 5)(2x + 5)**
3. Factor x4 – 49  
   (x2)2 = x4, 72 = 49,  
   **(1) (x2 – 7)( x2 + 7)**
4. Factor x4 + 7x2 + 12  
   Factors **(3,4)**, (-3,-4), (2,6), (-2, -6), (1,12),   
   (-1,-12)  
   **(3) (x2 + 3)(x2 + 4)**
5. Factor x4 – x2 – 20  
   Factors: **(4, -5)**, (-5, 4), (2, -10), (-2, 10),   
   (-1, 20), (1, -20)  
   x4 – x2 – 20 =   
   **(1) (x2 - 5)(x2 + 4)**
6. Factor 9x2 – 16y29 = 32, 16 = 42  
   **(2) (3x – 4y)(3x + 4y)**
7. Factor a4 + 10a2 + 25  
   25 = 52,   
   **(3) (a2 + 5)2**
8. Factor x6 – 4  
   x6 = (x3)2, 4 = 22  
   **(3) (x3 – 2)( x3 + 2)**
9. Factor (3x)2 + 8(3x) + 12  
   (1)   
   (3x + 4)(3x +3) = (3x)(3x)+(3x)(3)+(4)(3x)+12 =  
   9x2 + 9x + 12x + 12 = 9x2 + 21x + 12  
   (2) (3x + 6)(3x + 2) = 9x2 + 6x + 18x + 12 =   
   **(2) 9x2 + 24x + 12**
10. Factor x4 – 10x2 + 16  
    Factors: (4,4), (-4,-4), (8,2), **(-8,-2)**  
    **(1) (x2 – 8)(x2 – 2)**
11. Factor (2x + 1)2 – 9  
    9 = 32**(2) (2x + 1 – 3)(2x + 1 + 3)**
12. Show how you arrived at your answers.
13. Factor completely x4 – 81  
    Difference of perfect squares pattern  
    x4 = (x2)2, 81 = 92  
    **(x2 + 3) (x2 - 3)**
14. Factor completely x4 + 4x2 – 5  
    Factors: **(5, -1),** (-5, 1)  
    **(x2 + 5) (x2 – 1)**
15. Factor completely: x4 – 13x2 + 36  
    Factors: (1, 36), (-1, -36), (2, 18), (-2, -18),   
    (3, 12), (-3, -12), (4, 9), **(-4, -9),** (6,6), (-6,-6)  
    -4 + -9 = -13  
    **(x2 – 4)(x2 – 9)**
16. Factor (3x - 5)2 + 2(3x – 5) – 3.  
    (3x)(3x)+(3x)(-5)+(-5)(3x)+(-5)(-5)+6x – 10 – 3 =  
    9x2 -15x – 15x + 25 + 6x -13 = 9x2 - 24x + 12 =   
    (3x – 2)(3x – 6) =   
    **(3x – 2)(3)(x – 2)**  
    Factors: (-2, -6)  
    ck: (3x – 2)(3x – 6) = 9x2–18x–6x+12 =   
    9x2 -24x + 12
17. The polynomial a3 – b3 can be factored into   
    (a - b)(a2 + ab + b2). Use this factoring pattern to factor the polynomial x6 – 8.  
    a = x2, b = 2  
      
    (x2 – 2)(x4 + 2x2 + 22) =  
    (x2 – 2)(x4 + 2x2 + 4)  
      
    ck: (x2)(x4)+(x2)(2x2) + 4x2 - 2x4 - 4x2 + (-2)(4) =  
    x6 + 2x4 – 2x4 + 4x2 – 4x2 – 8 = x6 – 8. Ck.